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LIQUID FIN—A NEW DEVICE FOR HEAT-TRANSFER EQUIPMENT

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Abstract—The principle and the experimental results of a thermally superconductive device—the "liquid fin"—are presented. A comparison with the heat pipe is given.

NOMENCLATURE

a,	thermal diffusivity $[\mathrm{m^2 s^{-1}}]$;
(Bi),	Biot number;
F,	function of the Biot number;
g,	gravitational acceleration $[\mathrm{ms^{-2}}]$;
L,	half-length of the liquid fin [m];
(Pr),	Prandtl number;
R,	liquid-fin radius;
$(Ra)_L$,	Rayleigh number based on the liquid-fin half-length;
$(Ra)_R$,	Rayleigh number based on the liquid-fin radius;
ΔT,	temperature difference between the working fluids [deg];
z,	vertical coordinate [m];
α,	heat transfer coefficient $[\mathrm{Wm^{-2}\,deg^{-1}}]$;
β,	thermal expansion coefficient $[\mathrm{deg^{-1}}]$;
δ	length of the central (insulated) zone of the liquid fin [m];
λ,	heat conductivity of the liquid-fin substance $[\mathrm{Wm^{-1}\,deg^{-1}}]$;
λ_e,	effective heat conductivity $[\mathrm{Wm^{-1}\,deg^{-1}}]$;
λ_w,	heat conductivity of the liquid-fin pipe wall $[\mathrm{Wm^{-1}\,deg^{-1}}]$;
ν,	kinematic viscosity $[\mathrm{m^2 s^{-1}}]$.

Subscripts

1,	heated zone;
2,	cooled zone;
cr,	critical conditions.

1. INTRODUCTION

Some schemes of the liquid fin device, as proposed by the authors [1], are shown in Fig. 1. The principle of operation is as follows. Suppose that the heat exchanging fluids are separated mutually by a thick isolating wall (Fig. 1a) or even are flowing in separate channels (Fig. 1b). Heat transfer between the fluids may be created or augmented by means of a system of pipes or loops filled with liquid, forming "liquid fins" heated by the hot fluid situated below, and cooled in the upper section by the fluid to be heated. Thus in the liquid fin a longitudinal temperature gradient is established, and at a certain value of this gradient the limit of stability region of pure heat conduction is reached. From this limit upwards the natural convection in liquid fin begins to play a significant role. Convection currents in vertical pipes and circulations in loops increase the effective heat conductivity of the liquid fin to a high degree; values of several thousands W/m deg has been reached in experiments with liquid metals. Such values are not observed

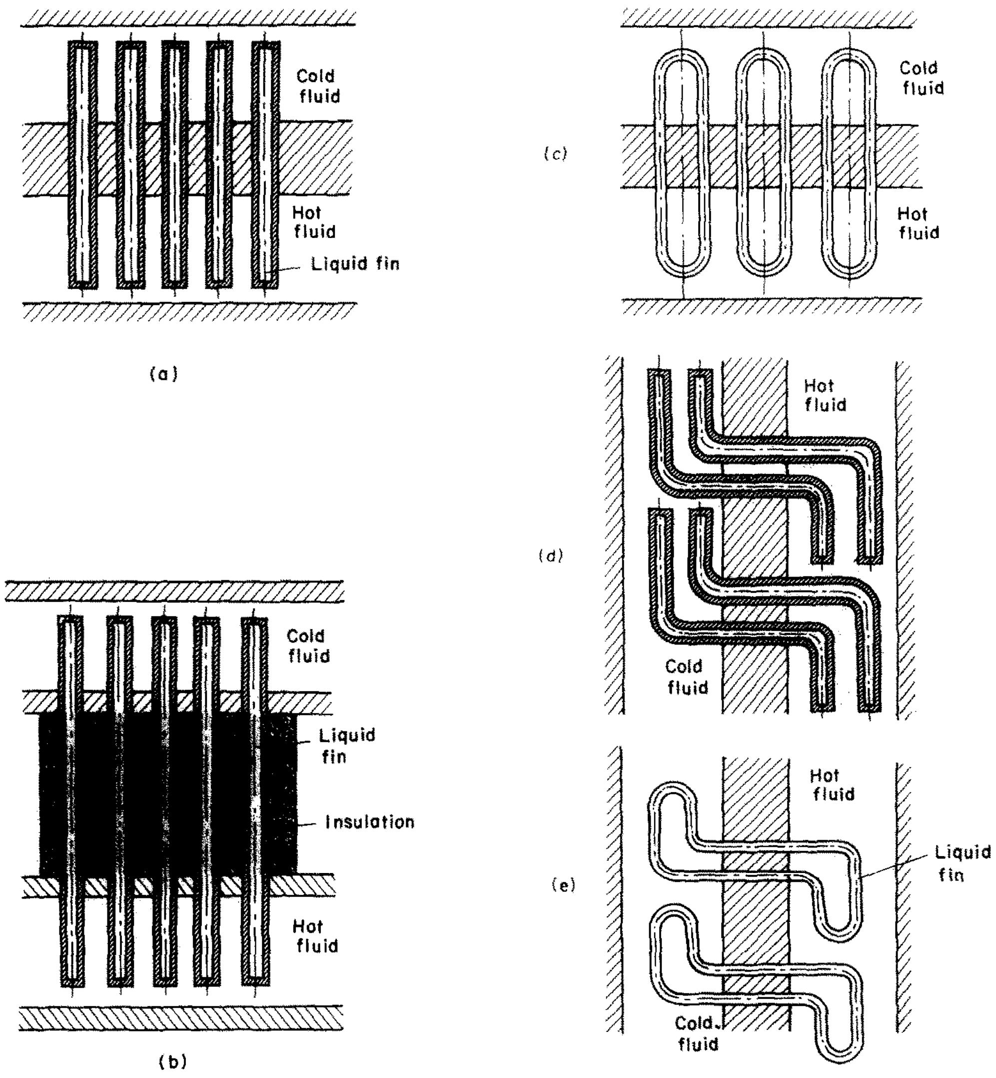

FIG. 1. Schemes of the liquid fin devices.

in nature. The result obtained is that the liquid fin as a superconductive device offers negligibly small thermal resistance in longitudinal direction. Therefore the heat transfer in heat exchangers of the type shown in Fig. 1 is governed primarily by convection between the fluids and the liquid fins.

The technical possibilities of the application of liquid fins are wide. The device may be applied, e.g. for high-temperature recuperators. With this application in view the liquid-fin substance should be a liquid metal not excluding other substances as molten salts. Therefore the first experiments to be reported below have been carried out using mercury and the NaK eutectic.

The experimental apparatus has been built according to the scheme from Fig. 1c (horizontal flow of fluids, lower fluid-hot, upper fluid-cold).

The vertical position of fluid-flow channels is also possible (see Fig. 1d, e). In this case the "active" pipe sections must be vertical, and the section in the hot fluid bent downwards, whereas the section in the cold fluid should be bent upwards. The same rule must be applied in horizontal flow recuperators with vertical separating wall.

2. THEORETICAL ANALYSIS

The first major theoretical problem to be solved is the determination of conditions at which natural convection currents are organized in the vertical liquid column (liquid fin). For a given temperature difference between the heat exchanging fluids ΔT the Rayleigh number

$$(Ra)_R = \frac{g\beta \Delta T R^3}{\nu a}$$

is given. The liquid fin is divided into three zones. The central zone is peripherically insulated; therefore the temperature gradient in this zone is constant. This is the maximum temperature gradient in the liquid fin, which is cooled in the upper zone, and heated in the lower zone. The intensity of heat transfer at these zones is determined by the heat-transfer coefficients α_1 (heated zone) and α_2 (cooled zone), or the Biot numbers $(Bi)_1 = \alpha_1 R/\lambda$ and $(Bi)_2 = \alpha_2 R/\lambda$ respectively.

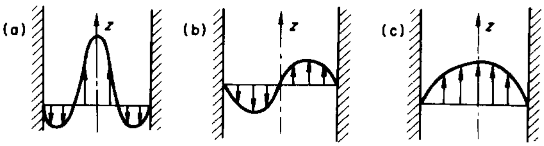

FIG. 2. Modes of liquid motion in the liquid fin.

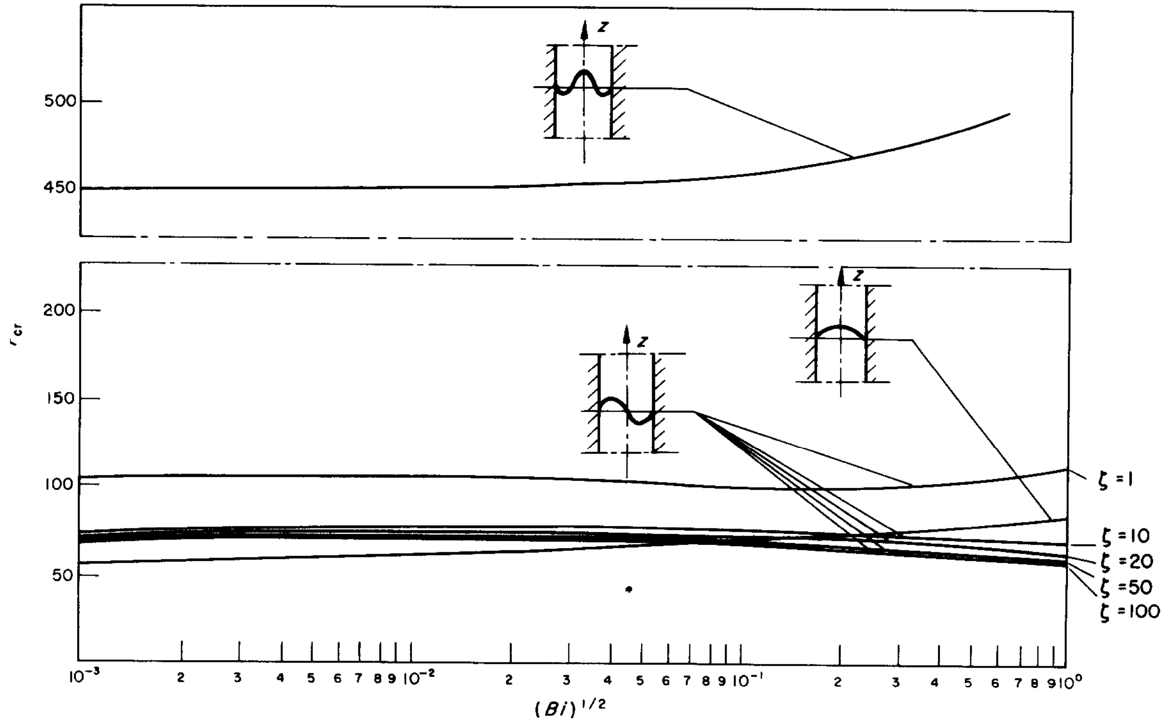

FIG. 3. The dependence of function F_{cr} upon the Biot number.

In both external zones the longitudinal

temperature gradient decreases in the direction to the ends of liquid fin. For small Rayleigh numbers it is therefore possible that the zone of convection currents comprises only a portion of the liquid fin (pipe). In such cases two fundamental modes of motion should be analysed, namely the symmetrical (Fig. 2a) and the antisymmetrical one (Fig. 2b). Using the Galerkin–Zhukhovitskii variational method [4], the function F_{cr}, depending upon the Biot number [$(Bi)_1$ for heated zone $(Bi)_2$ for cooled zone] has been found. The results are shown in Fig. 3. For the antisymmetrical case the function F depends also upon the ratio of heat conductivities $\zeta = \lambda/\lambda_w$, where λ_w is the conductivity of the pipe wall. Denoting the length of the central zone by δ, and the coordinate of a section in the external zones by z (z is measured upwards or downwards from one of the ends of the central zone), one may determine from the heat-transfer equation the value of

$$F = (Ra)_R \cdot \frac{1}{\frac{z}{R}(Bi)^{\frac{1}{2}}\left\{\frac{\delta}{R} + \frac{\operatorname{cth}[(Bi)_1^{\frac{1}{2}}\, L/R]}{(Bi)_1^{\frac{1}{2}}} + \frac{\operatorname{cth}[(Bi)_2^{\frac{1}{2}}\, L/R]}{(Bi)_2^{\frac{1}{2}}}\right\}} \times \frac{\operatorname{ch}\left[(Bi)^{\frac{1}{2}}\frac{L-z}{R}\right] - \operatorname{ch}\left[(Bi)^{\frac{1}{2}}\frac{L}{R}\right]}{\operatorname{sh}\left[(Bi)^{\frac{1}{2}}\frac{L}{R}\right]},$$

in which (Bi) stands for either $(Bi)_1$ or $(Bi)_2$. The condition for existence of convection currents is that $F > F_{cr}$. Thus if $(Bi)_1$, $(Bi)_2$, (Ra) and the geometry (δ, R, L) are given, one may calculate the limits z_1, z_2 of the convection zone by putting $F = F_{cr}$ If $z = L$ the currents spread upon whole liquid fin. In this case it must be

$$(Ra)_R > F_{cr} \cdot \frac{L}{R}(Bi)^{\frac{1}{2}}\left\{\frac{\delta}{R} + \frac{\operatorname{cth}\left[(Bi)_1^{\frac{1}{2}}\frac{L}{R}\right]}{(Bi)_1^{\frac{1}{2}}} + \frac{\operatorname{cth}[(Bi)_2^{\frac{1}{2}}]}{(Bi)_2^{\frac{1}{2}}}\right\} \cdot \frac{\operatorname{sh}\left[(Bi)^{\frac{1}{2}}\frac{L}{R}\right]}{1 - \operatorname{ch}\left[(Bi)^{\frac{1}{2}}\frac{L}{R}\right]}$$

for either $(Bi) = (Bi)_1$ or $(Bi) = (Bi)_2$.

In the case of loops (Fig. 1c, e) the velocity profile, shown in Fig. 2c, is also possible. In this case liquid circulates through the pipe loop. The results for this case are also shown in Fig. 3, which is reproduced from [2], by Mikielewicz.

The second theoretical problem is the calculation of the mean temperature difference for the countercurrent parallel flow recuperator of the type shown in Fig. 1. This analysis, performed in [2], uses finite differences method. The results indicate that for more than 10 liquid-fin pipes in file the log-mean temperature difference is to be recommended. For a smaller number of pipes the mean temperature difference is somewhat higher but does not exceed the arithmetic mean.

3. EXPERIMENTAL RESULTS

Experiments have been carried out on a metallic recuperator working on the principle from Fig. 1c. The heat exchanging fluids, namely air and combustion gases, were fed by use of the same compressor with combustion chamber as the heat source. The scheme of the experimental installation is shown in Fig. 4. Internal separation wall of the recuperator with the liquid-fin loop section is removable. Photographs of these liquid-finned walls are shown in Figs. 5 and 6. The wall has been made from asbestos sheets, and the loops from stainless steel pipes, filled with mercury or the NaK eutectic. From measurements, performed at relatively small Reynolds numbers based on liquid-fin pipe diameter (300–4000 for high-temperature fluid, 200–2000 for low-temperature fluid), the temperature distribution in liquid fins, and the heat transfer coefficients α_1 and α_2 have been determined. On evaluating the heat flux in liquid fins it has been possible to determine the effective heat conductivity λ_e of the liquid fins. The ratio

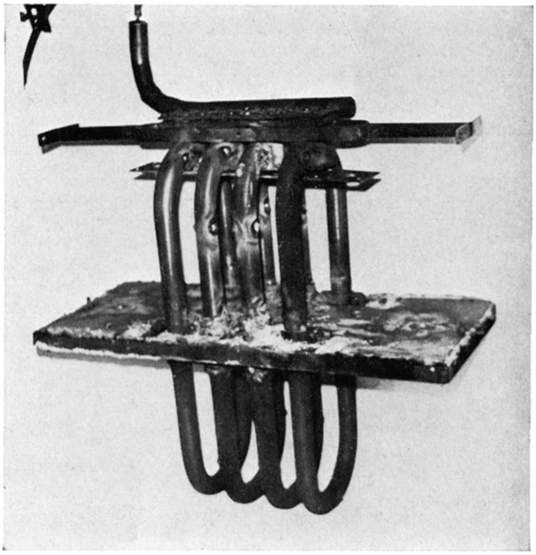

FIG. 5. Wall of the experimental recuperator with liquid fins.

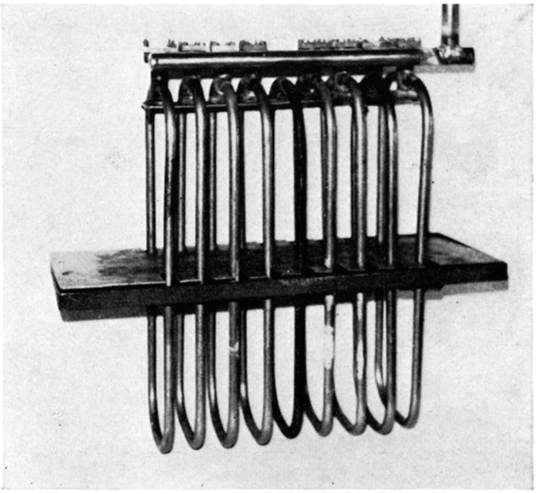

FIG. 6. Wall of the experimental recuperator with liquid fins—second version.

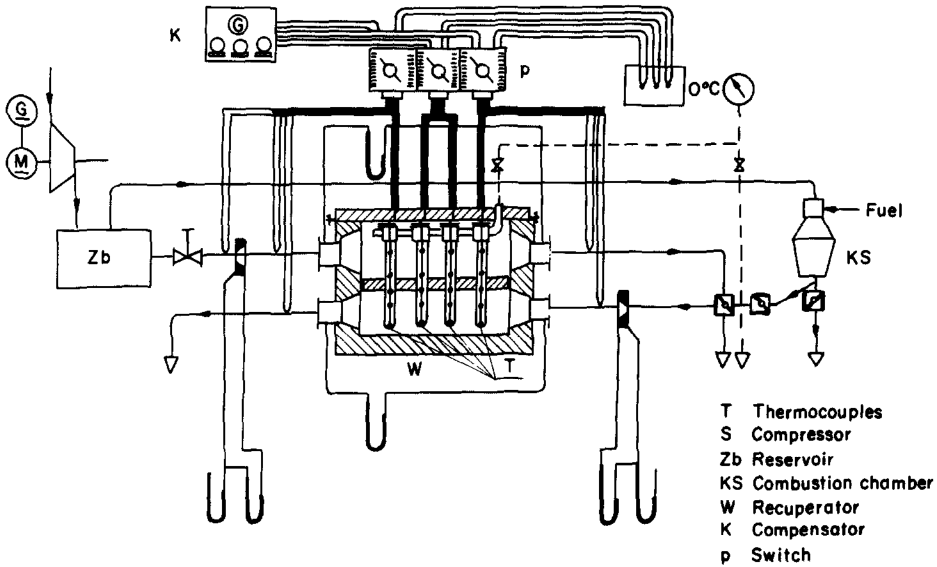

FIG. 4. The scheme of the experimental installation.

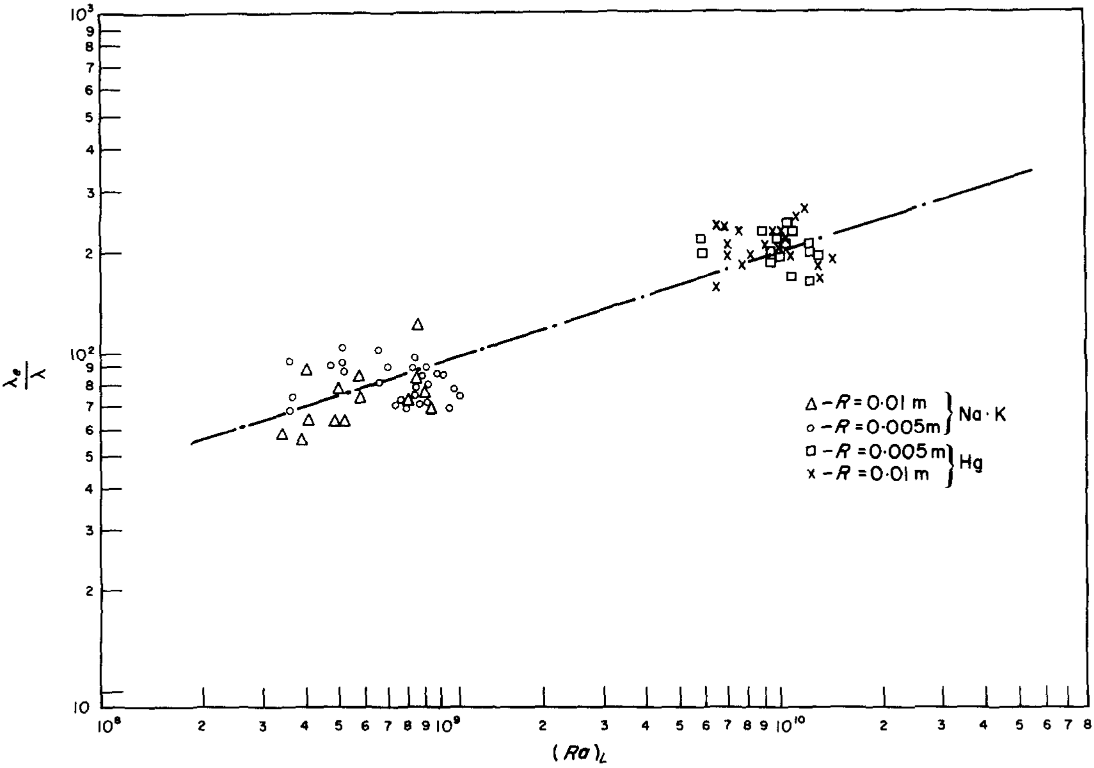

FIG. 7. The ratio λ_e/λ v. $(Ra)_L$ obtained from experiments for liquid fins.

λ_e/λ vs. $(Ra)_L = g\beta\Delta TL^3/\nu a$ (with L as the half-length of the liquid fin) is shown in Fig. 7 for ratios $\alpha_1/\alpha_2 = 0{\cdot}44$–$0{\cdot}62$, obtained in experiments. The dashed line represents the Silveston correlation [6] for horizontal enclosed fluid spaces with width L:

$$\frac{\lambda_e}{\lambda} = 0{\cdot}10(Pr)^{0{\cdot}05}\,.\,(Ra)_L^{0{\cdot}31}.$$

The agreement of both results seems to be curious.

4. COMPARISON OF THE LIQUID-FIN DEVICE WITH THE HEAT PIPE

It has been shown that the liquid fin is a thermally superconductive device. Another device with such property is the heat pipe invented 1944 by Gangler [7]. The principle of the heat pipe is discussed, e.g. in [5]. This device consists of two tubes, the inner tube being of a porous material, closed on both ends and filled with working fluid which evaporates in the heated zone and condenses in the cooled zone of the pipe. The condensed liquid is transported in porous wall by means of capillarity forces from the cooled zone to the heated zone. This motion corresponds to the convection currents in the liquid fin.

On comparing the two superconductive devices the major advantage of the heat pipe should be first pointed out. Namely this device does not require the gravitational field, because the liquid motion is governed by capillarity forces. This feature is very important for application at zero-gravity. On the contrary, the liquid fin cannot operate at zero-gravity, lest in the case of artificial force field. On the other hand, the liquid fin is much simpler (a single tube instead of composed one). The pressure of the liquid fin is independent of temperature range, whereas in the heat pipe the pressure is governed by temperature. If the substance for the liquid fin is properly chosen, such restrictions as the critical heat flux in the heat pipe are not actual for liquid fins.

The proper choice of substances for liquid fins is based on the following requirements:

Liquid state in the whole temperature range,
Good thermal conductivity,
Great value of the material function $\beta/\nu a$ to obtain great Rayleigh numbers,
High saturation temperature at working pressure,
Possibly low temperature of solidification.

These conditions are fulfilled by liquid metals as aluminum, gallium, tin, lead with the application to high-temperature heat transfer equipment in view.

It is evident that a direct comparison of liquid fin with heat pipe is not possible on the base of the performed experiments, e.g. the results, quoted in [5] have been obtained at high thermal fluxes in heat pipes, and the effective heat conductivities have reached the values of the order 10^6 W/m.deg. In the experiments with liquid fins reported here the thermal fluxes were much smaller because of poor heat transfer from the gases to the liquid fin. The effective heat conductivities have been of the order 10^3 W/m.deg. These figures have been—however—quite sufficient to obtain a negligibly small temperature gradient in liquid fins.

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"AILETTE LIQUIDE"—UNE CONCEPTION NOUVELLE D'ÉQUIPEMENT POUR TRANSFERT THERMIQUE

Résumé—On présente le principe et les résultats expérimentaux sur la réalisation d'un élément thermiquement supraconducteur: "l'ailette liquide". On établit une comparaison avec les tubes chauffés.

FLÜSSIGE RIPPE-EINE NEUE VORRICHTUNG FÜR DEN WÄRMEÜBERGANG

Zusammenfassung—Es werden das Prinzip und die Versuchsergebnisse angegeben für eine thermisch supraleitende Vorrichtung die "flüssige Rippe".

Ein Vergleich mit der "heat pipe" wird gemacht.

НОВОЕ ТЕПЛООБМЕННОЕ УСТРОЙСТВО

Аннотация—В статье одисывается принцип действия и представлены результаты экспериментального исследования разработанного автором сверхпроводящего устройства. Дается сравнение с тепловой трубкой.